Surface Expressions of Nonlinear Internal Waves

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LONG-TERM GOALS

To provide a more detailed understanding of the surface roughness and radar backscatter variations associated with nonlinear internal waves under a variety of environmental conditions.

OBJECTIVES

To determine the effects of wave-current interactions and nonlinear surface wave interactions on the development of the short wave spectrum in the presence of oceanic internal waves.

APPROACH

Our approach is to compare the results of direct numerical simulations (Choi) with predictions using the wave action equation (Lyzenga), and to update the wave action equation as necessary in order to account for nonlinear effects. Direct numerical simulations are being done using the approach of Choi (1995) as modified to include variable surface currents. We are evaluating the incorporation of a nonlinear energy transfer term into the wave action equation using the diffusion approximation as described by Zakharov and Pushkarev (1999) and Polnikov *et al*. (2002). Results will be compared with available laboratory measurements, synthetic aperture radar and shipboard radar data collected during the SCS05 experiment, and video imagery collected during the SW06 experiment.

WORK COMPLETED

Last year, we carried out a number of direct numerical simulations showing the evolution of a 1-D surface wave field in the presence of a variable surface current similar to that induced by an internal wave. We compared these numerical results with predictions of the wave action equation in the absence of external forcing, and found generally good agreement.

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This year, a parallel code was implemented in order to extend the numerical simulations to two (horizontal) dimensions. The wave action model was also extended to 2-D and modified to include external forcing and nonlinear energy transfer effects, using a diffusion approximation for the nonlinear energy transfer term. Some initial numerical simulations were done to investigate the cascade of wave energy to shorter wavelengths, and additional simulations are planned to validate the diffusion approximation. We also participated in the SW06 experiment by mounting and operating two video cameras aboard the R/V Endeavor. A large amount (377 GB) of video data was recorded, showing surface wave perturbations including wave breaking in the presence of internal waves.

RESULTS

Solutions of the wave action equation without nonlinear energy transfer show large perturbations of meter-scale surface waves in the presence of internal wave-induced surface currents (Lyzenga and Bennett, 1988; Lyzenga, 1998). There seems to be a general consensus that these changes can filter down to the shorter (centimeter-scale) wavelengths that more strongly influence the microwave radar backscatter, but an accurate and efficient means of accounting for this cascade mechanism has not yet been devised. We believe that the aforementioned diffusion approximation may provide an approach to the solution of this problem. In the formulation of Polnikov *et al*. (2002), this approximation is incorporated by adding a term of the form

$$S_{nl} = \frac{a}{\omega g^4}\left[\frac{\omega^2}{2}\frac{\partial^2 y}{\partial \omega^2} + \frac{\partial^2 y}{\partial \phi^2}\right]$$

to the right-hand side of the wave growth equation, where $y = \left[\omega^4 S(\omega,\phi)\right]^3$, $S(\omega,\phi)$ is the wave height spectrum, and *a* is a constant on the order of 0.1. Following Zakharov and Pushkarev (1999), the growth equation

$$\frac{\partial S}{\partial t} = \beta S + \frac{a}{\omega g^4}\left[\frac{\omega^2}{2}\frac{\partial^2 y}{\partial \omega^2} + \frac{\partial^2 y}{\partial \phi^2}\right]$$

is re-written as

$$\frac{\partial y}{\partial t} = 3\beta y + \frac{3a\omega^3}{g^4}\left[\frac{\omega^2}{2}\frac{\partial^2 y}{\partial \omega^2} + \frac{\partial^2 y}{\partial \phi^2}\right] y^{2/3}$$

and this equation is solved numerically for *y* using a partially implicit differencing scheme. The result is then transformed into the dimensionless function

$$B(k,\phi) = \frac{k^3}{\omega^4}\left(\frac{\partial \omega}{\partial k}\right) y^{1/3}$$

which is equivalent to *k*4 times the wave height spectrum expressed in wavenumber coordinates. An example solution illustrating the cascade effect introduced by the nonlinear term is shown in Figure 1. In this case, a narrow Gaussian function centered at a frequency of 1.5 Hz is used for β. Initially, the wave spectrum follows the shape of this energy input term, but when *B* reaches a level on the order of 10^{-3} the spectrum begins to broaden rapidly toward higher wavenumbers.

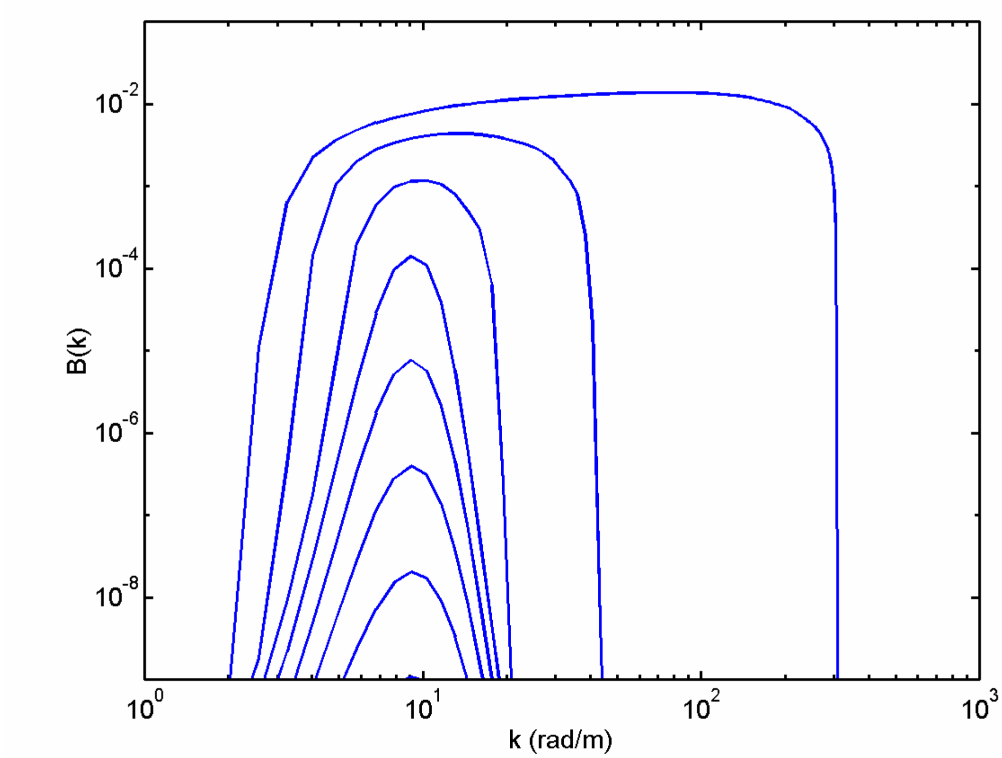

Figure 1. Solution of the wave growth equation for a Gaussian-shaped energy input term centered at a frequency of 1.5 Hz. Curves indicate values of $B(k,\phi)$ ***averaged over azimuth angles, at 30 second time intervals.***

Our next step will be to incorporate this term into the full wave action equation, examine the properties of the solutions in the presence of surface currents induced by internal waves, and compare the results with direct numerical simulations and experimental measurements.

As part of this project, we also took the opportunity of participating at a relatively low level in the SW06 experiment by placing a pair of video cameras on the bow mast of R/V Endeavor. The cameras were mounted at an elevation of 10.2 meters above the water surface, with a look angle of 45° from the vertical and a field of view of approximately 20° for the camera on the left side and 60° for the camera on the right, as shown in Figure 2. A few selected frames from the camera with the smaller field of view are shown in Figure 3, along with the 2-D image spectra computed from these frames. These images were collected while the ship was crossing an internal wave packet on August 9, 2006, and illustrate the large changes in the surface wave patterns that occur due to interactions with the internal wave surface currents. Future analyses may include estimates of the amount of small-scale wave breaking, and possibly measurements of surface currents by tracking waves and surface foam.

IMPACT/APPLICATIONS

The results of this investigation are expected to improve the quality and quantity of information that can be extracted from radar and visible images of oceanic internal waves.

Figure 2. Photograph of video cameras mounted on the bow mast of the R/V Endeavor.

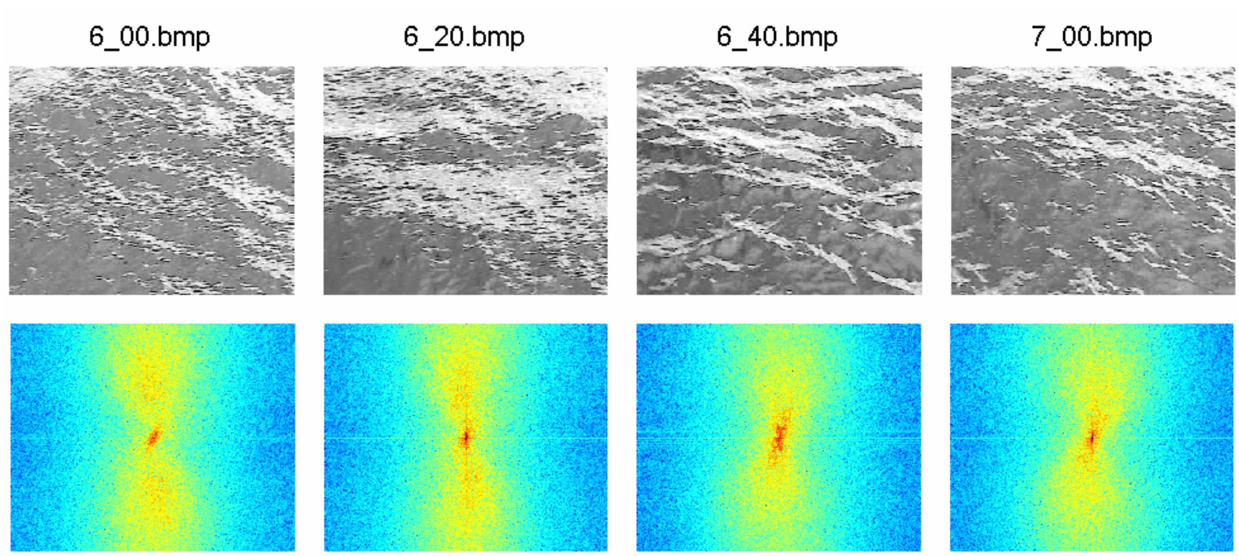

Figure 3. Sample video frames selected at 20-second time intervals during passage of an internal wave packet on August 9, 2006 (original data is recorded at 30 frames per second).

RELATED PROJECTS

This investigation is related to a number of other projects funded by the ONR Non-Linear Internal Wave Initiative (NLIWI). Specifically, we plan to use the surface currents from the internal wave modeling efforts conducted as part of this initiative, and will compare our predictions with remote sensing data collected during the NLIWI field experiments.

The co-investigators on this project are also co-investigators on the University of Michigan MURI project on Optimum Vessel Performance in Evolving Nonlinear Wave Fields. This project involves some related issues in the areas of surface wave and radar backscatter modeling.

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